

About the **incompressible energy cascade** rate in **anisotropic solar wind turbulence**

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ABSTRACT

Context. The presence of a magnetic guide field induces several types of anisotropy in solar wind turbulence. The energy cascade rate between scales in the inertial range depends strongly on the direction of this magnetic guide field, splitting the energy cascade according to the parallel and perpendicular directions with respect to magnetic guide field.

Aims. Using more than 2 years of Parker Solar Probe (PSP) observations, the isotropy and anisotropy energy cascade rates are investigated. The variance and spectral anisotropy ratios, the kinetic and magnetic energies and the both normalized cross-helicity and residual energy are studied. The connection between the heliocentric distance, the local temperature of the plasma and the energy cascade components is made.

Methods. Using exact relations for fully developed magnetohydrodynamic (MHD) turbulence, the incompressible energy cascade rate is computed. In particular, using the isotropy (Politano & Pouquet 1998a,b) and 2D and slab (MacBride et al. 2008) assumptions, the isotropic, perpendicular and parallel energy cascade rate components are estimated.

Results. The variance anisotropy ratios, for both kinetic and magnetic fields, do not exhibit a dependence with respect to the heliocentric distance, r . While the kinetic spectral anisotropy ratio shows a dependence with r , the magnetic spectral anisotropy does not. A strong correlation between the isotropic and anisotropic energy cascade rates and the temperature is found. A clear dominance of the perpendicular cascades over the parallel cascades as PSP approaches to the Sun is observed. A dominant 2D cascade/geometry over the slab component in slow solar wind turbulence in the largest MHD scales is observed.

1. Introduction

The solar wind expansion from the Sun is highly non-adiabatic, partly noticed by proton temperatures falling off much more slowly than what is expected for a freely expanding ideal gas (e.g., Parker 1958; Richardson et al. 1995). Throughout its radial expansion, the solar wind develops a strongly turbulent regime (Bruno & Carbone 2005), which can be characterized by proton density, velocity, temperature and magnetic field fluctuations (Matthaeus & Velli 2011). Furthermore, large-scale magnetohydrodynamic (MHD) turbulence serves as a reservoir of energy that cascades down to the smallest scales, where it can be dissipated by kinetic effects while it heats the plasma (e.g., Leamon et al. 1998; Sahraoui et al. 2009; Alexandrova et al. 2009). In the MHD inertial range, where the energy is transferred without dissipation through different spatial and temporal scales (e.g., Frisch 1995), the solar wind exhibits a constant energy cascade rate as a function of such scales (Sorriso-Valvo et al. 2007; Coburn et al. 2015; Hadid et al. 2017; Bandyopadhyay et al. 2020; Andrés et al. 2021), in which the magnetic spectrum presents a $-5/3$ slope (e.g., Matthaeus & Goldstein 1982; Leamon et al. 1998; Matthaeus 2021).

The presence of a magnetic guide field \mathbf{B}_0 induces several types of anisotropy in solar wind turbulence in the MHD and kinetic scales (see, Horbury et al. 2012). In particular, the energy transfer between scales depends

strongly on the direction of the magnetic guide field, splitting the energy cascade according to the parallel and the perpendicular directions with respect to \mathbf{B}_0 . Several observational results have shown that the solar wind fluctuations at 1 astronomical unit (au) at the largest MHD spatial scales are a combination of field-aligned (or slab) and perpendicular (or 2D) wavevectors (see, Matthaeus et al. 1990; Dasso et al. 2005). Dasso et al. (2005) used 5 years of ACE data from near-Earth orbit to investigate the correlation anisotropy of solar wind MHD scale fluctuations and showed that the nature of the anisotropy differs in fast and slow solar winds. In particular, fast winds are relatively more dominated by fluctuations with wavevectors quasi-parallel to the local magnetic field, while slow solar winds, which appear to be more fully evolved turbulence, are more dominated by quasi-perpendicular fluctuation wavevectors. Adhikari et al. (2021) studied anisotropic turbulence in the slow and fast solar wind as a function of the angle between the mean solar wind speed and the mean magnetic field and as a function of the heliocentric distance. Using Solar Orbiter measurements, the authors compared the observed results with the solar wind plus nearly incompressible MHD turbulence transport model equations (Zank & Matthaeus 1993), and found agreement between the theoretical and observed results in the slow and fast winds as a function of the heliocentric distance.

Typically, there are two types of fluctuation anisotropy that are recurrently observed in the solar wind, spectral and variance anisotropy (see, Oughton et al. 2015). On one hand, if the components of the fluctuating magnetic (or kinetic) field have unequal energies, then the field is said to exhibit variance or component anisotropy (Matthaeus et al. 2005; Weygand et al. 2011). On the other hand, when the energy distribution at a given spatial (i.e., ℓ) or temporal (i.e., τ) scale is not isotropic, one speaks of spectral or wavevector anisotropy (Montgomery & Turner 1981; Shebalin et al. 1983; Goldreich & Sridhar 1995; Oughton et al. 2015). In the present paper, we focus our attention in two particular features of anisotropic turbulence, the variance anisotropy ratio and the ratio between fluctuation and mean field for the velocity and the magnetic fields, respectively. The investigation of these anisotropy ratios, the energy cascade rate in the MHD scales, the isotropic and anisotropic models and its connection with the solar wind temperature is the main objective of the present paper.

Using exact relations in fully developed turbulence is possible to obtain expressions for the energy cascade rate. Assuming spatial homogeneity and full isotropy, an exact relation for incompressible MHD turbulence can be derived (Politano & Pouquet 1998b,a). This exact relation provides a precise computation of the amount of energy per unit time and volume ε_1 (or heating rate) as a function of the velocity and magnetic correlation functions. The MHD exact relation and its connection with the nonlinear energy cascade rate has been numerically and observationally validated for both incompressible and compressible MHD turbulence (Weygand et al. 2007; Matthaeus et al. 1999; Grossmann et al. 1997; Carbone et al. 2009; Stawarz et al. 2009, 2010; Banerjee et al. 2016; Hadid et al. 2017, 2018; Andrés et al. 2018; Andrés & Banerjee 2019), and has been generalized to include sub-ion scale effects (Andrés et al. 2018; Andrés et al. 2019; Hellinger et al. 2018; Ferrand et al. 2019, 2021a). Estimations of the isotropic energy cascade rate in the inertial range of solar wind turbulence have been previously computed at 1 au (see, Marino et al. 2008; Coburn et al. 2015; Banerjee et al. 2016; Hadid et al. 2017) and more recently at small and large heliocentric distances (see, Bandyopadhyay et al. 2020; Andrés et al. 2021).

Assuming a 2D and slab cylindrical symmetric geometry, where one has to assume that the perpendicular cascade rate depends only on the perpendicular scale and the parallel cascade depends on the parallel direction, MacBride et al. (2008) derived a relation for homogeneous incompressible anisotropic MHD turbulence. In particular, they derived expressions for the correlation functions that are applicable to both parallel and perpendicular cascades. Using 7 years of solar wind observations from the ACE spacecraft at 1 au, MacBride et al. (2008) found a linear scaling of the energy flux, as is expected for the MHD inertial range. Also, they found that both fast and slow solar winds exhibit an active energy cascade over a inertial range, with an energy cascade rate in the parallel direction consistently smaller than in the perpendicular direction. Stawarz et al. (2009) investigated the convergence of third-order structure functions to compute cascade rates in the solar wind using ACE observation at 1 au covering the years from 1998 till 2007. The authors found that a minimum of one year of data is normally required to get good convergence and statistically significant results. Also, they compared the computed energy cascade rates with previously determined rates of proton heating at 1 au as determined from the radial gradient of the proton temperature. Stawarz et al. (2010) investigated ACE observations of large cross-helicity states using isotropic and anisotropic expression for the energy cascade rate. In contrast to intervals with small helicity values, large helicity states demonstrate a significant back-transfer of energy from small to large scales.

In the present paper, using a large PSP data set (more than 5000 hours in the solar wind), we extend the current state of knowledge of solar wind turbulence in the inner heliosphere by computing the energy cascade rate using both the anisotropic and isotropic relations for fully developed turbulence. Using magnetic field and plasma moments observations between ~ 0.2 au and ~ 0.8 au, we investigate how the energy cascade rate is affected not only by the heliocentric distance, but also by the presence of a guide magnetic field and the consequence anisotropy. The study is structured as follows: in Sections 2 and 3, we present the theoretical incompressible MHD model and a brief description of the anisotropic and isotropic exact relations, respectively. In Section 4 we briefly describe the PSP observation data set and the conditions that each turbulent event must fulfill. In Sections 5 we present the main results of our analysis. Finally, the discussion and conclusions are developed in Section 6.

2. The incompressible MHD model

The three-dimensional (3D) incompressible MHD equations are the momentum equation for the velocity field \mathbf{u} (in which the Lorentz force is included), the induction equation for the magnetic field \mathbf{B} , and the solenoid condition for both fields. These equations can be written as,

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{B} \cdot \nabla \mathbf{B} - \frac{1}{\rho_0} \nabla (P + P_M) + \mathbf{f}_k + \mathbf{d}_k, \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{B} + \mathbf{B} \cdot \nabla \mathbf{u} + \mathbf{f}_m + \mathbf{d}_m, \quad (2)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (4)$$

where the magnetic field is in Alfvén velocity units, i.e., the real magnetic field is $\mathbf{B} \sqrt{4\pi\rho_0}$ (where ρ_0 is the mean mass density and μ is the magnetic permeability of the plasma) and P_M is the magnetic pressure. Finally, $\mathbf{f}_{k,m}$ are respectively a mechanical and the curl of the electromotive large-scale forcings, and $\mathbf{d}_{k,m}$ are respectively the small-scale kinetic and magnetic dissipation terms (Andrés et al. 2016; Ferrand et al. 2021b).

3. The exact relation in MHD turbulence

Using Eq. (1)-(4) and following the usual assumptions for fully developed homogeneous turbulence (i.e., infinite kinetic and magnetic Reynolds numbers and a steady state with a balance between forcing and dissipation (see, e.g. Andrés & Sahraoui 2017), an exact relation for incompressible anisotropic MHD turbulence can be obtained as (e.g., Galtier 2018),

$$-4\varepsilon = \rho_0 \nabla_\ell \cdot \mathbf{F}, \quad (5)$$

where \mathbf{F} is the so-called incompressible energy flux,

$$\mathbf{F} = \rho_0 \langle (\delta \mathbf{u} \cdot \delta \mathbf{u} + \delta \mathbf{B} \cdot \delta \mathbf{B}) \delta \mathbf{u} - (\delta \mathbf{u} \cdot \delta \mathbf{B} + \delta \mathbf{B} \cdot \delta \mathbf{u}) \delta \mathbf{B} \rangle, \quad (6)$$

and ε is the total energy cascade rate per unit volume. Fields are evaluated at position \mathbf{x} or $\mathbf{x}' = \mathbf{x} + \boldsymbol{\ell}$; in the latter case a prime is added to the field. The angular bracket $\langle \cdot \rangle$ denotes an ensemble average (Batchelor 1953), which is taken here as time average assuming ergodicity. Finally, we have introduced the usual increments definition, i.e., $\delta \alpha \equiv \alpha' - \alpha$. It is worth mentioning that we do not have access to multi-spacecraft measurements, and therefore, it is mandatory to assume some sort of symmetry to integrate Eq. (5) and be able to compute the energy cascade rate ε (see, Stawarz et al. 2011). In particular, we shall work with two models for the energy cascade rate, an isotropic form for ε_I (Politano & Pouquet 1998a,b) and two anisotropic expressions ε_\perp and ε_\parallel for the perpendicular and parallel cascade rates (MacBride et al. 2008), respectively.

3.1. The isotropic energy cascade rate (Politano & Pouquet 1998a,b)

Assuming the Taylor hypothesis (i.e., $\ell \equiv \tau U_0$, where U_0 is the mean plasma flow speed and $\ell = |\boldsymbol{\ell}|$ is the longitudinal distance) and fully isotropy, Eq. (5) can be integrated and expressed as a function of time lags τ . While Eq. (5) includes increments in all the spatial directions, the isotropic cascade only includes

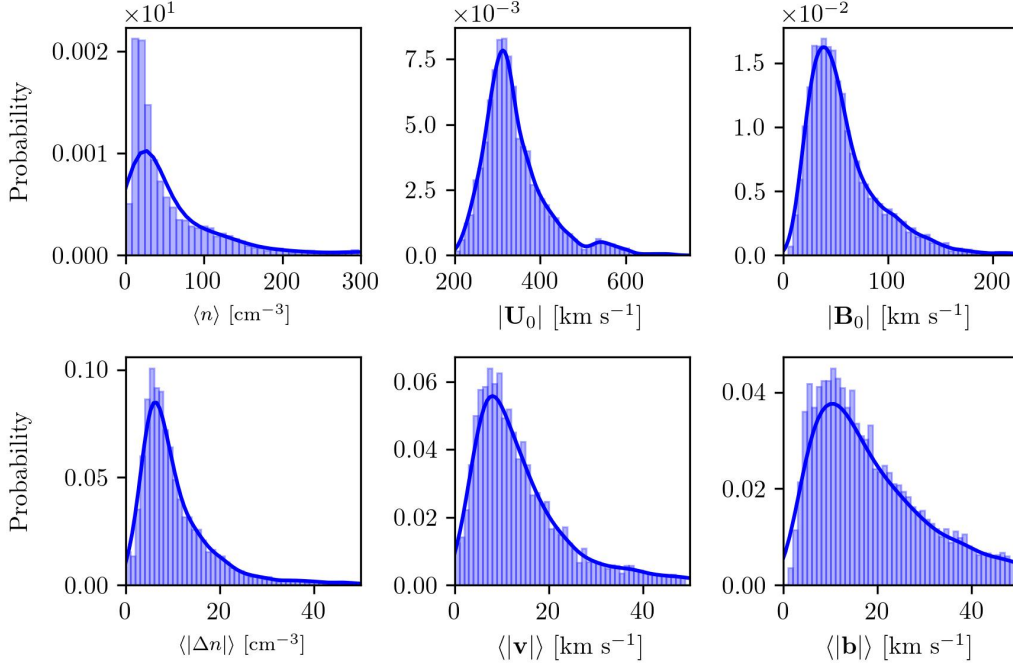


Fig. 1. The occurrence rate for the proton density, the proton and Alfvén velocity absolute mean values, in the first row and fluctuations in the second row, respectively.

increments in the longitudinal direction ℓ (for single-spacecraft measurements, in the plasma velocity direction $\hat{\mathbf{U}}_0$). Therefore, the isotropic energy cascade rate can be computed as,

$$\varepsilon_I = \rho_0 \langle [(\delta \mathbf{u} \cdot \delta \mathbf{u} + \delta \mathbf{B} \cdot \delta \mathbf{B}) \delta u_\ell - (\delta \mathbf{u} \cdot \delta \mathbf{B} + \delta \mathbf{B} \cdot \delta \mathbf{u}) \delta B_\ell] / (-4\tau U_0/3) \rangle. \quad (7)$$

where $u_\ell = \mathbf{u} \cdot \hat{\mathbf{U}}_0$ and $B_\ell = \mathbf{B} \cdot \hat{\mathbf{U}}_0$. In particular, the total isotropic energy cascade rate ε_I can be expressed as a function of two components, ε_1 proportional δu_ℓ , and another ε_2 proportional to δB_ℓ .

3.2. The 2D and slab energy cascade rates (MacBride et al. 2008)

As we discussed in the Introduction, observational results have shown that energy power is confined to the parallel and perpendicular directions with respect to the magnetic guide field (e.g., Shebalin et al. 1983; Dasso et al. 2005; Oughton et al. 2013a). Therefore, here we present the hybrid formulation (i.e., 1D plus 2D) that can address the parallel and perpendicular fluctuations, temporal increments, and energy cascade rates (see, MacBride et al. 2008; Stawarz et al. 2009). To find expressions for the perpendicular and parallel cascade rates, we use the magnetic field basis (e.g., Bieber et al. 1996), where the velocity and magnetic fields observations are properly rotated to leave parallel magnetic fluctuations in one direction. Then, in this particular basis, the $\hat{\mathbf{e}}_3$ versor is along the magnetic guide field direction and the unit vectors are,

$$\hat{\mathbf{e}}_3 \equiv \hat{\mathbf{e}}_B, \quad (8)$$

$$\hat{\mathbf{e}}_2 \equiv \hat{\mathbf{e}}_3 \times \hat{\mathbf{e}}_1, \quad (9)$$

$$\hat{\mathbf{e}}_1 \equiv \frac{\hat{\mathbf{e}}_U \times \hat{\mathbf{e}}_B}{|\hat{\mathbf{e}}_U \times \hat{\mathbf{e}}_B|}, \quad (10)$$

where $\hat{\mathbf{e}}_B = \langle \mathbf{B} \rangle / \langle |\mathbf{B}| \rangle$ and $\hat{\mathbf{e}}_U = \langle \mathbf{u} \rangle / \langle |\mathbf{u}| \rangle$. Assuming that we have cylindrical symmetry and the energy flux (6) is perpendicular to the mean magnetic field (and depends only on ℓ_\perp), an expression for the perpendicular energy cascade rate can be found as,

$$\varepsilon_\perp = \rho_0 \langle [(\delta \mathbf{u} \cdot \delta \mathbf{u} + \delta \mathbf{B} \cdot \delta \mathbf{B}) \delta u_2 - (\delta \mathbf{u} \cdot \delta \mathbf{B} + \delta \mathbf{B} \cdot \delta \mathbf{u}) \delta B_2] / (-2\tau U_0 \sin \theta_{BV}) \rangle \quad (11)$$

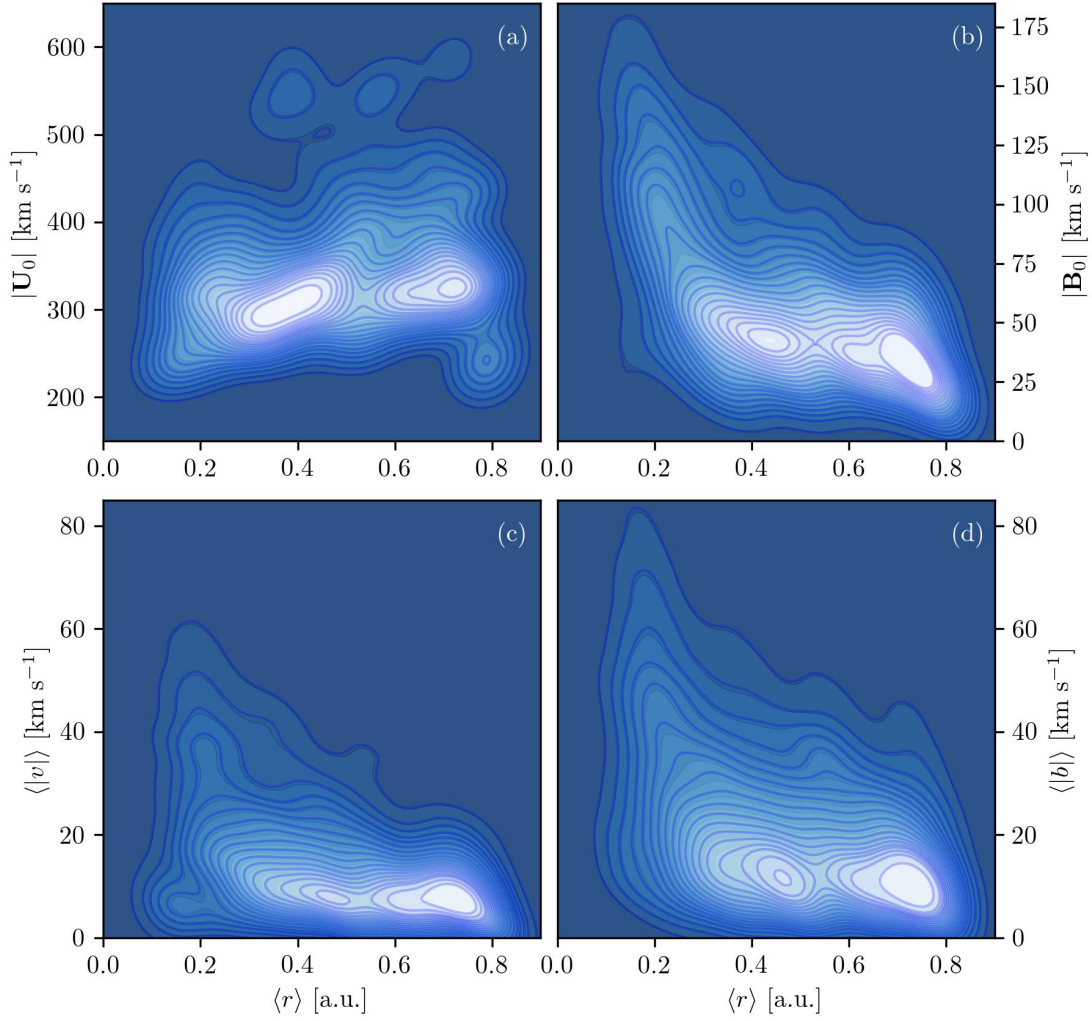


Fig. 2. Bi-variant kernel density estimation (KDE) for the mean ((a) and (b)) and fluctuating ((c) and (d)) kinetic and magnetic field absolute values as a function of the heliocentric distance, respectively.

where $u_2 = \mathbf{u} \cdot \hat{\mathbf{e}}_2$, $B_2 = \mathbf{B} \cdot \hat{\mathbf{e}}_2$ and θ_{BV} is the angle between $\hat{\mathbf{e}}_B$ and $\hat{\mathbf{e}}_U$. On the other hand, still assuming that we have cylindrical symmetry but the energy flux (6) is parallel to mean magnetic field and depends only in the parallel direction ℓ_{\parallel} , an expression for the parallel cascade rate can be found as,

$$\varepsilon_{\parallel} = \rho_0 \langle [(\delta \mathbf{u} \cdot \delta \mathbf{u} + \delta \mathbf{B} \cdot \delta \mathbf{B}) \delta u_3 - (\delta \mathbf{u} \cdot \delta \mathbf{B} + \delta \mathbf{B} \cdot \delta \mathbf{u}) \delta B_3] / (-4\tau U_0 \cos \theta_{BV}) \rangle. \quad (12)$$

where $u_3 = \mathbf{u} \cdot \hat{\mathbf{e}}_3$ and $B_3 = \mathbf{B} \cdot \hat{\mathbf{e}}_3$. Finally, the total hybrid energy cascade rate in this model is $\varepsilon_H = \varepsilon_{\perp}/2 + \varepsilon_{\parallel}/4$. In the present paper, we are interested in compute ε_{\perp} , ε_{\parallel} and ε_H , which are fully defined by velocity and magnetic field increments that can be estimated from single in-situ measurements.

4. Observations and selection criteria

We used a data set of PSP observations (Fox et al. 2016; Kasper et al. 2016; Bale et al. 2016; Kasper et al. 2019; Bale et al. 2019; Case et al. 2020) covering the period between October 10, 2018 and December 31, 2020. This large data set includes the first six PSP's perihelia. We have used the magnetic field and the proton moments from the FIELDS and SPC experiments, respectively. The spurious data (i.e., high artificial peaks) in the SPC moments (see, Kasper et al. 2016) were removed using a linear interpolation (see, Bandyopadhyay et al. 2020; Parashar et al. 2020) and the data set was re-sampled to 0.873 s time resolution. In order to analyze the solar wind turbulence at the MHD scales, the data set were divided into a series of samples of equal duration of 60 minutes, respectively. This time duration ensures having several correlation time of the turbulent fluctuations at heliocentric distances less than 1 au (see, Parashar et al. 2020; Hadid et al. 2017).

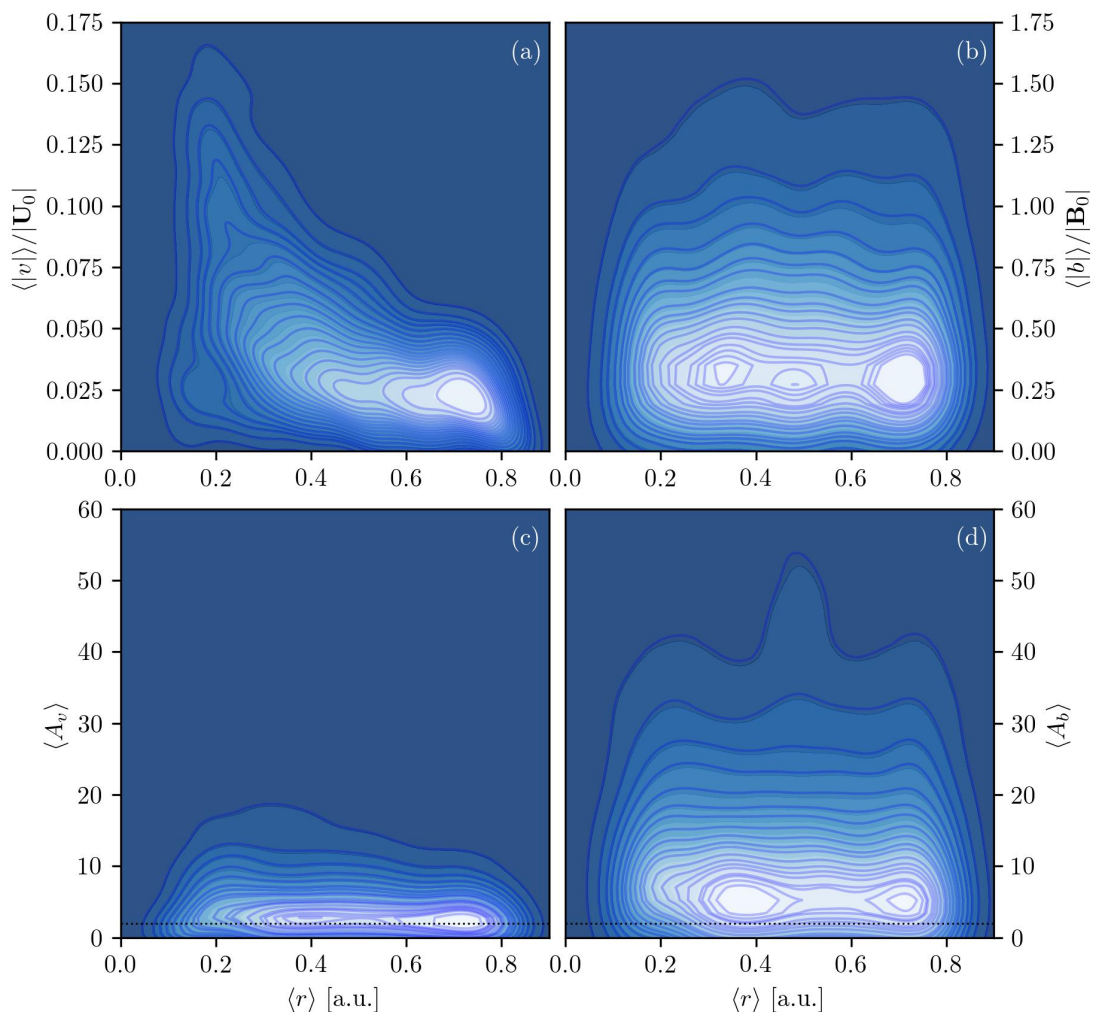


Fig. 3. Bi-variant KDE for the spectral anisotropy ((a) and (b)) and variance anisotropy ((c) and (d)) ratios for the kinetic and magnetic fields as a function of the heliocentric distance, respectively. The dot lines in (c) and (d) correspond to the isotropic (kinetic or magnetic) energy distribution.

As in previous studies (e.g., Andrés et al. 2020; Andrés et al. 2021), we avoided intervals that contained significant disturbances or large-scale gradients (e.g., coronal mass ejection or interplanetary shocks) or rapid flips in the Sun’s magnetic field that reversed direction (i.e., magnetic switchbacks). We further considered only intervals that did not show large fluctuations of the energy cascade rate over the MHD scales, typically we retained events with $\text{std}(\varepsilon_1)/\text{mean}(|\varepsilon_1|) < 1$ (where std is for standard deviation).

5. Results

5.1. Occurrence rates

Figure 1 shows the occurrence rate for the number density, velocity, and magnetic field absolute mean and fluctuation values, respectively, for all the events in our data set. In particular, we have separated the velocity and magnetic fields in terms of its mean and fluctuation values as,

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{U}_0 + \mathbf{v}(\mathbf{x}, t), \quad (13)$$

$$\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0 + \mathbf{b}(\mathbf{x}, t). \quad (14)$$

where $\mathbf{U}_0 = \langle \mathbf{u}(\mathbf{x}, t) \rangle$, $\mathbf{B}_0 = \langle \mathbf{B}(\mathbf{x}, t) \rangle$ and $\langle \dots \rangle$ denotes a time averaging operator, which in the present paper is the global mean (i.e., a one hour average). It is worth mentioning that most of the cases studied in the present paper correspond to slow solar wind (i.e., $|\mathbf{U}_0| \lesssim 500 \text{ km s}^{-1}$). Since we want to investigate the incompressible energy cascade rates, to ensure the incompressibility assumption, we shall keep only the cases

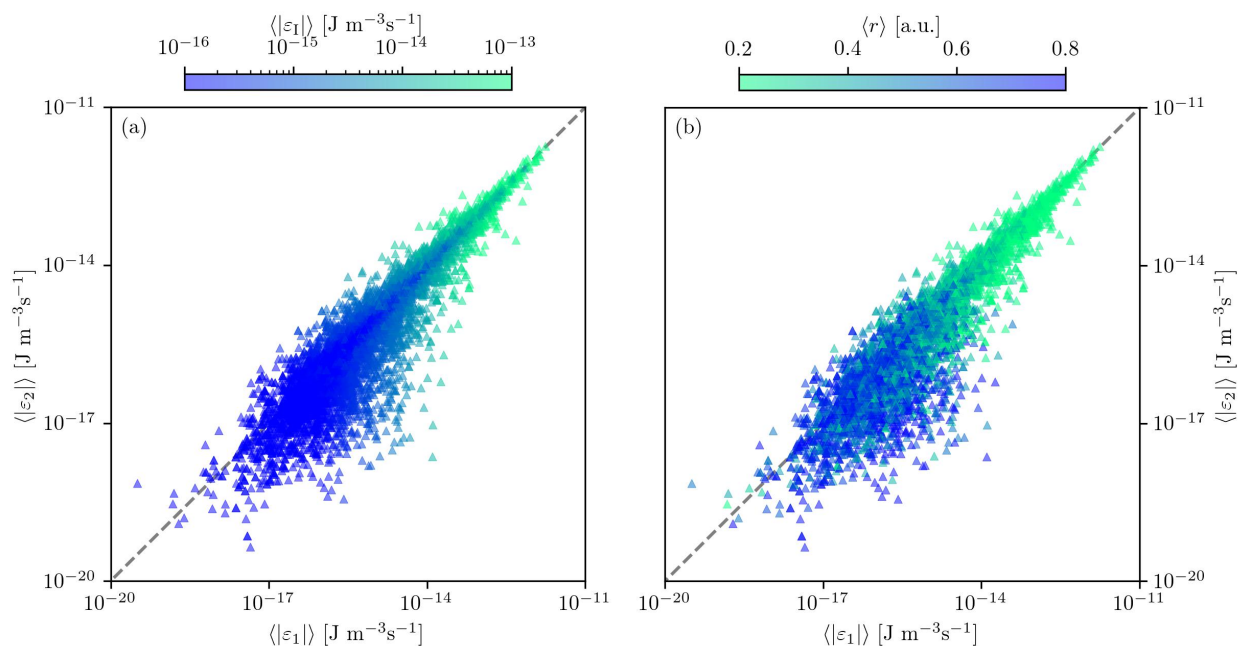


Fig. 4. (a) Cascade rate component $\langle \varepsilon_2 \rangle$ as a function of the component $\langle \varepsilon_1 \rangle$. In panel (a), the colorbar is the total cascade $\langle \varepsilon_1 \rangle$ and in panel (b), the heliocentric distance $\langle r \rangle$.

where $\langle \Delta n/n \rangle < 15\%$ (where $\Delta n \equiv n - \langle n \rangle$). This leave us with a data set of ~ 5200 event of 1 hour duration each.

Figure 2 shows the bi-variant Kernel Density Estimation (KDE) for the mean ((a) and (b)) and fluctuating ((c) and (d)) velocity and magnetic fields as a function of the heliocentric distance, respectively. A bi-variant KDE produces a continuous probability density surface in two dimensions (see, Waskom 2021), where brighter regions correspond to regions with more analyzed events. As a typical histogram, a bi-variant KDE shows a particular data set as an interpretative plot. It is worth noting that while the mean velocity field values do not present a statistical dependence with the heliocentric distance, the magnetic guide field and both magnetic and kinetic fluctuation values strongly decrease as we move away from the Sun. In particular, as we approach to the Sun, the magnetic and kinetic fluctuation levels increase up to the same order ($\sim 50 - 70 \text{ km s}^{-1}$). We will return to this point in Section 5.3 when we analyzed the isotropic cascade rate.

5.2. Variance and spectral anisotropy ratios

As we discussed in the Introduction, there are two types of fluctuation anisotropy that are typically observed in the solar wind, spectral anisotropy and variance anisotropy. To quantify them, we consider the velocity and magnetic fields in terms of mean values plus fluctuations around these means (see Eqs. (13) and (14)). On the one hand, if the components of the field have unequal energies (e.g., in cartesian coordinates, departures from $\langle b_x^2 \rangle = \langle b_y^2 \rangle = \langle b_z^2 \rangle$ for the magnetic field), the field exhibit variance anisotropy (e.g., TenBarge et al. 2012). To quantify this variance anisotropy, we consider the kinetic and magnetic anisotropy ratios (see, Oughton et al. 2015) as,

$$A_v = \frac{v_{\perp}^2}{v_{\parallel}^2}, \quad (15)$$

$$A_b = \frac{b_{\perp}^2}{b_{\parallel}^2}, \quad (16)$$

where we have employed the magnetic field coordinate system (see, Bieber et al. 1996). Variance anisotropy is scale dependent (e.g., Matthaeus et al. 2012), however, in the present paper we focus our attention in their values for the largest MHD scales (i.e, one hour mean values). On the other hand, generally speaking, when the energy distribution at a given time scale τ is not isotropic, we speak of spectral anisotropy. In particular,

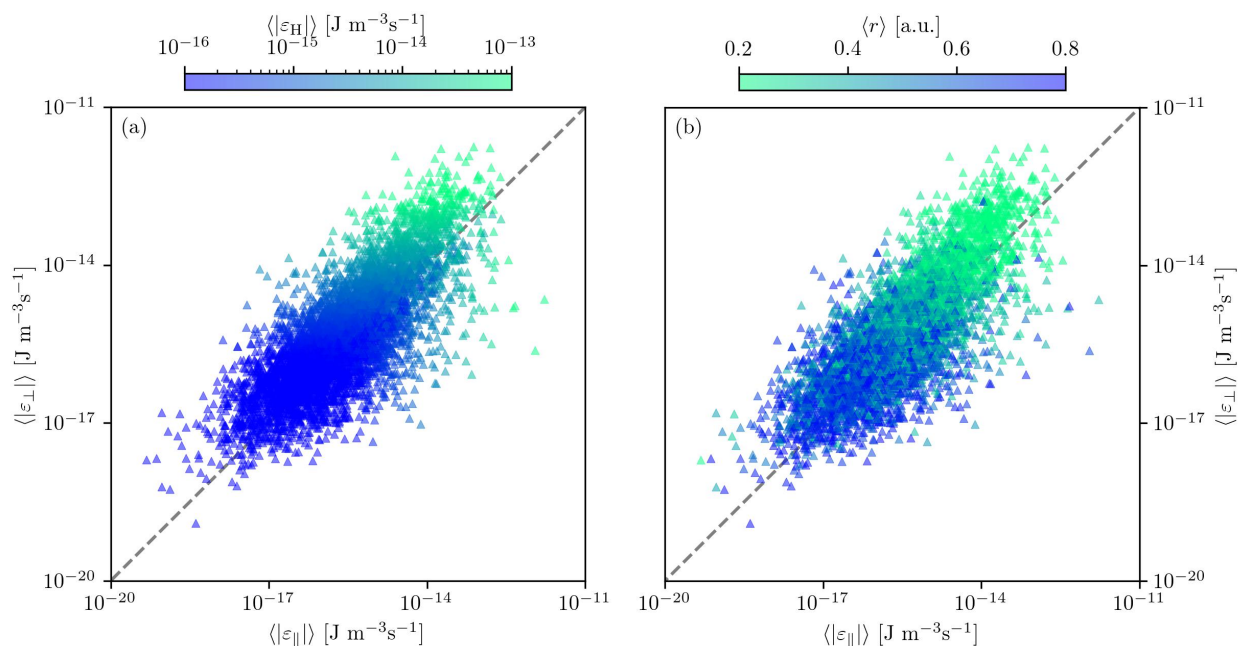


Fig. 5. (a) Cascade rate component $\langle |\varepsilon_{\perp}| \rangle$ as a function of the component $\langle |\varepsilon_{\parallel}| \rangle$. In panel (a), the colorbar is the total cascade $\langle |\varepsilon_H| \rangle$ and in panel (b), the heliocentric distance $\langle r \rangle$.

spectral anisotropy is usually associated with energy cascades that are also anisotropy (Oughton et al. 2015; Horbury et al. 2012). Moreover, for incompressible MHD turbulence, numerical and observational evidence showed that strong (or even moderate) mean magnetic fields give rise to a suppression of the energy cascade in the parallel direction and, therefore, the perpendicular energy cascade is much stronger than the parallel cascade (Milano et al. 2001; MacBride et al. 2008; Stawarz et al. 2009; Oughton et al. 2013b; Matthaeus et al. 2012; Andrés et al. 2018). Therefore, in the present paper we consider the ratio between the mean and the fluctuation fields as an indicative of spectral anisotropy at the MHD scales for both \mathbf{u} and \mathbf{B} , i.e., the ratios $\langle |v| \rangle / U_0$ and $\langle |b| \rangle / B_0$ are the spectral anisotropy for the kinetic and magnetic fields, respectively.

Figure 3 show the bi-variant KDE for the spectral anisotropy ratios ((a) and (b)) and variance anisotropy ratios ((c) and (d)) for the kinetic and magnetic fields as a function of the heliocentric distance, respectively. The dot lines in Figures 3 (c) and (d) correspond to the isotropic (kinetic or magnetic) energy distribution. While the kinetic spectral anisotropy ratios show a dependence with the heliocentric distance r (with a very low amplitude), the magnetic spectral anisotropy does not show a clear dependence. However, the magnetic fluctuations are much larger than their means, while the kinetic fluctuation are small when they are compared with their means. The variance anisotropy ratios, both kinetic and magnetic, do not exhibit a dependence with respect to the heliocentric distance. Moreover, for the velocity field most of the cases remain around 2 implying that the kinetic energy distribution is approximately isotropic in the MHD scales and for the magnetic field most of the events reported here show large anisotropy ratios (i.e., $2 \leq A_b$).

5.3. The incompressible energy cascade rate

To compute the right hand side of Eqs. (7), (11) and (12), we constructed temporal correlation functions of the different turbulent fields at different time lags τ in the interval [1,3600] s, which allows covering the MHD inertial range (Hadid et al. 2017). Once we have the energy cascade rates as a function of the time increments, we average them in the large time scales, i.e., for $\tau \in [1000, 3000]$ s to obtain representative values for the cascades in the largest MHD scales.

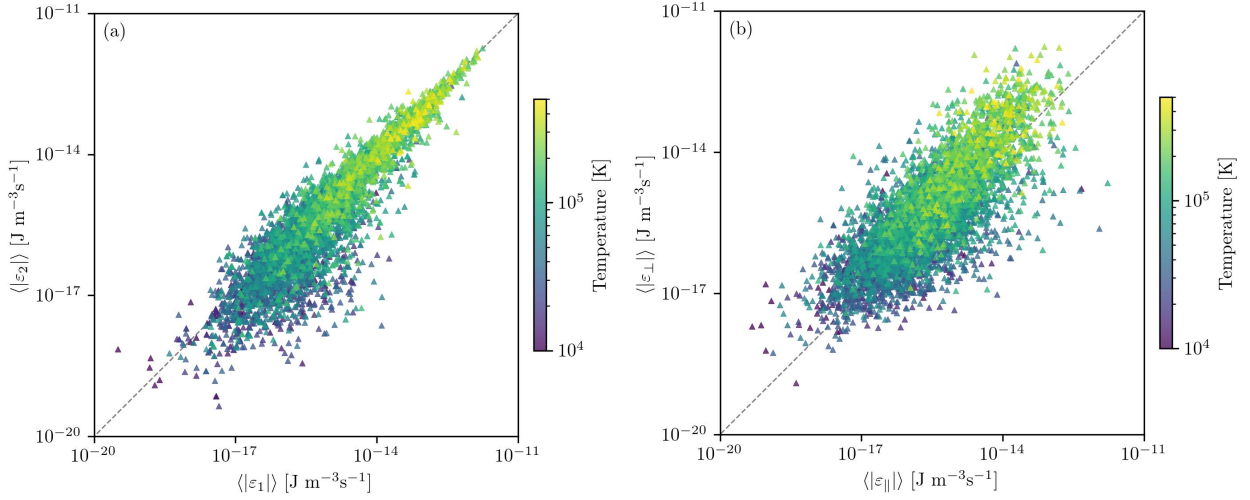


Fig. 6. Cascade rate component $\langle \varepsilon_2 \rangle$ as a function of the component $\langle \varepsilon_1 \rangle$ and the perpendicular component $\langle \varepsilon_{\perp} \rangle$ as a function of the parallel component $\langle \varepsilon_{\parallel} \rangle$. In both panels (a) and (b), the colorbar is the temperature.

As we discussed in Section 3.1, the total isotropic energy cascade rate can be written as a function of two components,

$$\varepsilon_1 = \varepsilon_1 + \varepsilon_2 \quad (17)$$

$$\varepsilon_1 = \rho_0 \langle (\delta \mathbf{u} \cdot \delta \mathbf{u} + \delta \mathbf{B} \cdot \delta \mathbf{B}) \delta u_{\ell} / (-4\tau U_0/3) \rangle. \quad (18)$$

$$\varepsilon_2 = -\rho_0 \langle (\delta \mathbf{u} \cdot \delta \mathbf{B} + \delta \mathbf{B} \cdot \delta \mathbf{u}) \delta B_{\ell} / (-4\tau U_0/3) \rangle, \quad (19)$$

where we can relate the first component ε_1 to the total (kinetic plus magnetic) energy and the second component ε_2 to the cross-helicity (i.e., $\mathbf{u} \cdot \mathbf{B}$) in the plasma. This interpretation comes directly from Eqs. (18) and (19), respectively.

Figure 4 shows the mean absolute value $\langle |\varepsilon_2| \rangle$ as a function of $\langle |\varepsilon_1| \rangle$. The color bars correspond to (a) the mean total energy cascade rate absolute value $\langle |\varepsilon_1| \rangle$ and (b) the heliocentric distance r , respectively. As a reference, we plot a gray dash line with slope equal to 1. As we expected, there is a strong correlation between the cascade rate amplitude and the heliocentric distance to the Sun, the closer is PSP to the Sun, the more strong is the isotropic energy cascade rate. In particular, the strongest cases correspond to equal cross-helicity and energy components (i.e., $\langle |\varepsilon_1| \rangle = \langle |\varepsilon_2| \rangle$).

Figure 5 shows the mean absolute value $\langle |\varepsilon_{\perp}| \rangle$ as a function of $\langle |\varepsilon_{\parallel}| \rangle$ in the same format that Figure 4. As in Figure 4, as we moved far away from the Sun, both components decrease their amplitudes. Moreover, we observe a clear trend to obtain more perpendicular than parallel energy cascade values as we approach to the Sun (slope larger than one in Figure 5 (b)).

5.4. The isotropic, perpendicular and parallel cascade rates and its relation with the temperature

Figure 6 shows the mean absolute value $\langle |\varepsilon_2| \rangle$ as a function of $\langle |\varepsilon_1| \rangle$ and the mean absolute value $\langle |\varepsilon_{\perp}| \rangle$ as a function of $\langle |\varepsilon_{\parallel}| \rangle$, respectively. In both panels, the colorbar correspond to the proton temperature and as a reference we add a gray dash line with slope equal to one. When we compared with Figures 4 and 5 we noted the clear (and expect) correlation between the heliocentric distance and the temperature, as r increases the temperature decreases. In the case of the anisotropic cascade rates, we also observed that the hottest events mainly correspond to those where the perpendicular cascade is dominant with respect to the parallel cascade in the MHD range.

Typically, in MHD it defines the normalized cross-helicity $\sigma_c = \langle \mathbf{v} \cdot \mathbf{b} \rangle / (E_k + E_m)$ and the normalized residual energy $\sigma_r = (\langle \mathbf{v}^2 \rangle - \langle \mathbf{b}^2 \rangle) / (\langle \mathbf{v}^2 \rangle + \langle \mathbf{b}^2 \rangle)$, where $E_k \equiv \langle \mathbf{v}^2 \rangle / 2$ is the incompressible kinetic energy and $E_m \equiv \langle \mathbf{b}^2 \rangle / 2$ is the magnetic energy. While the cross-helicity measures the level of Alfvénicity of a particular event, the residual energy quantifies the relative energy in kinetic and magnetic fluctuations. By definition, both parameters σ_c and σ_r ranges between -1 and 1. For simplicity we shall drop the “normalized” prefix, assuming the understanding that these imply the normalized versions σ_c and σ_r . Figure 7 shows the

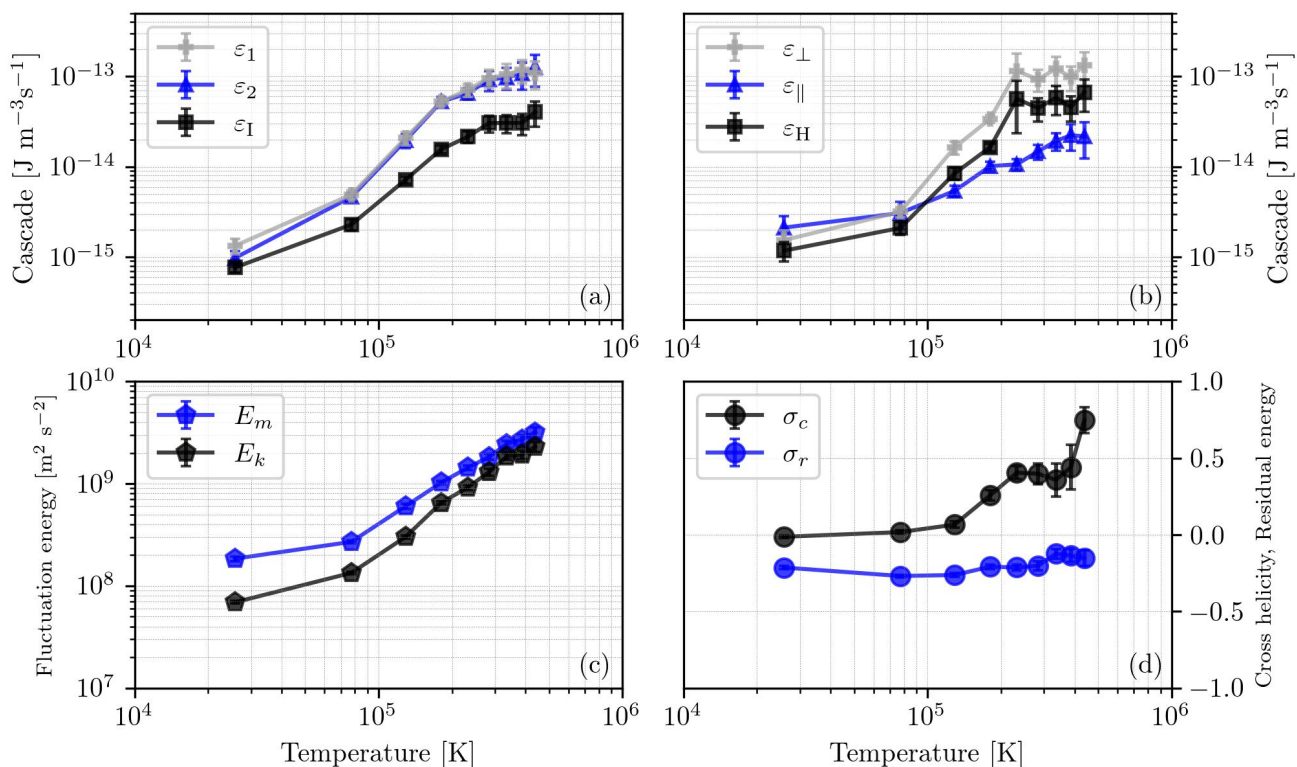


Fig. 7. (a) Components and total isotropic energy cascades rates; (b) components and total anisotropic energy cascades rates; (c) fluctuation kinetic and magnetic energies and (d) normalized cross-helicity and normalized residual energy as a function of the temperature, respectively.

average of different variables as a function of the temperature. In particular, we group events according to the temperature values and then bin average them. The error bars correspond to the standard deviation divided by the square root number of samples in each group. Then, for a given temperature, we have averaged (a) the isotropy and (b) anisotropy energy cascade rates (total and components), (c) the incompressible kinetic and magnetic fluctuation energies and (d) the cross-helicity and residual energy, respectively.

Figure 7 (a) and (b) show in a compact form the results analyzed in the previous Figure 6, as the isotropic (or anisotropic) energy cascade rate increases the temperature increases in the plasma. In particular, for the isotropic cascade the events with the largest temperatures correspond to $\langle |\varepsilon_1| \rangle = \langle |\varepsilon_2| \rangle$, while for the anisotropic cascade these events correspond to $\langle |\varepsilon_\perp| \rangle > \langle |\varepsilon_\parallel| \rangle$. Interestingly, in these hottest events the kinetic and magnetic fluctuation energies become approximately equal. Moreover, these events are almost Alfvénic events since $\sigma_c \rightarrow 1$.

6. Discussion and conclusions

In the present paper, we analyzed a large PSP solar wind data set of ~ 5200 events, covering observations from October 2018 until December 2020. Our statistical results show a clear correlation between the incompressible energy cascade rate, heliocentric distance and plasma temperature in the inner heliosphere. In particular, for both isotropic and anisotropic rates, as we decrease the heliocentric distance, the energy cascade rates increase several orders of magnitude. We have covered heliocentric distance from ~ 0.8 au up to ~ 0.1 au, obtaining energy cascade rates from $\sim 1 \times 10^{-19} \text{ J m}^{-2} \text{ s}^{-1}$ up to $\sim 1 \times 10^{-12} \text{ J m}^{-2} \text{ s}^{-1}$. Recently, Bandyopadhyay et al. (2020) estimated the isotropic energy cascade rate for the first PSP perihelion. The authors found that ε_I at ~ 0.17 au is about 100 times higher than the average value at 1 au. In agreement with this finding and previous statistical results (see, MacBride et al. 2008; Andrés et al. 2021), we have found an amplification of ε_I and ε_H as we approach to the Sun. This amplification as we decrease the heliocentric distance is due to the increase in the kinetic and magnetic fluctuation amplitudes (see Figure 2) and the mean solar wind density value.

In contrast with previous results (Oughton et al. 2015), we do not observe a clear dependence of the spectral and variance anisotropy ratios with the heliocentric distance in the inner heliosphere. Oughton et al. (2015)

reported a review about solar wind anisotropy with different anisotropy ratios A_v and A_b from slow and fast solar wind at different heliocentric distances. Bruno et al. (1999) have computed A_v and A_b for 3 events at 0.3, 0.7 and 0.9, respectively. The authors found that magnetic fluctuation variance ratio slightly increase with heliocentric distance, while the kinetic ratio remains constant. On the other hand, using Helios 1 observations from 0.3 au to 1 au, MacBride et al. (2010) showed that the magnetic variance anisotropy scales with both proton beta and the amplitude of fluctuation power spectrum with no dependence with the heliocentric distance. In agreement with MacBride et al. (2010), our statistical results do not show any apparent increase in A_b (or A_v) with respect to the heliocentric distance. Moreover, we observe that most of the cases exhibit $A_b > A_v$ (see, Bruno et al. 1999) confirming previous results (Oughton et al. 2015).

Using the isotropic assumption (Politano & Pouquet 1998a,b) and the slab and 2D assumption (MacBride et al. 2008), we computed the energy cascade rate components from both models. For the isotropic model, in the cases near the Sun (i.e., the largest cascade values or most hot events) both energy and cross-helicity components (see Eqs. (1) and (2)) are approximately equals. On the contrary, for the anisotropic model, in the same events the dominant component is the perpendicular one. At 1 au, using ACE solar wind observations from 1998 to 2005, MacBride et al. (2008) reported different cascade values for different types of solar wind. The authors found that both fast and slow solar wind exhibit an active cascade rate over the inertial range and the energy flux in the parallel cascade is consistently smaller than in the perpendicular cascade. Beyond the fact that we are exploring different heliocentric distances at different correlation times (an independent event last 2 days for MacBride et al., while for us an event last 1 hour) we observed the same trend: for a large majority of the cases the perpendicular cascade is much larger than the parallel one. This statistical result is totally consistent with a dominant 2D cascade/geometry in slow solar wind turbulence in the MHD scales (e.g., Shebalin et al. 1983; Matthaeus et al. 1996; Dasso et al. 2005; Wan et al. 2012; Oughton et al. 2013b; Andrés et al. 2017; Brodiano et al. 2021; Zank et al. 2021). Moreover, the nearly incompressible (NI) MHD model (e.g., Zank & Matthaeus 1993; Zank et al. 2021) predicts that the energy-containing range in the slow solar wind is a superposition of a majority quasi-2D component and a minority slab component. Recently, using the NI model, PSP observations and Solar Orbiter observations, Zank et al. (2021) and Adhikari et al. (2021) have shown that both the slow and fast solar wind is not typically aligned with large-scale magnetic field, and therefore, the quasi-2D fluctuations are visible to the PSP spacecraft, in agreement with our findings here.

We found a robust correlation between the temperature, the heliocentric distance and the isotropic and anisotropic energy cascade rates: as we approach to the Sun both temperature and cascade rates increase. The temperature rise is clearly related to the most Alfvénic events ($\sigma_c \rightarrow 1$) in a imbalanced ($E_m > E_k$) and magnetic fluctuations dominant regime ($\sigma_r < 0$). Using a NI MHD model, Zank et al. (2021) predicted arbitrary values of the (normalized) residual energy with a tendency to evolve toward negative values in magnetic energy dominated regimes. The authors also analyzed PSP slow solar wind observations showing that the normalized residual energy becomes increasingly negative with increasing heliocentric distance, i.e., becoming magnetic energy-dominated with distance. In the present paper we confirm these predictions, exploring not only the heliocentric distance dependence but also the amplification of the cascade and the local temperature. While, we do not observe that σ_r becomes increasingly negative with increasing heliocentric distance, we do observe a constant and negative value for σ_r as we approach to the Sun. Also, these observations about σ_c and σ_r are consistent with the dominant 2D structures over the minority slab component.

Finally, some aspects of these work require improvement. On one hand, we do not take into account possible compressible under various closures (Simon & Sahraoui 2021a,b), which may be relevant even in the usual incompressible solar wind (Banerjee et al. 2016; Hadid et al. 2017; Andrés et al. 2017; Andrés et al. 2021). On the other hand, we did not include the sub-ion scales energy cascade physics (Andrés et al. 2018; Andrés et al. 2019; Hellinger et al. 2018; Ferrand et al. 2021a), which are intimately related to the solar wind heating problem (e.g., Matthaeus 2021). These issues are planned for the upcoming works.

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